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Physics 35100 Mechanics
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Problem Set 1

From *Classical Mechanics*, R. Douglas Gregory:
Chapter 1: 1.1, 1.3, 1.14, 1.17 (optional) [1.7, 1.10-13]
Chapter 2: 2.1, 2.12 (optional) [2.6, 2.20]
Chapter 3: (optional) [3.1 3.6]

Question 1. Any triangle can be represented by three vectors $\vec{u}$, $\vec{v}$, $\vec{w}$.

1) Make a sketch of an example triangle with the tails of $\vec{u}$ and $\vec{v}$ at the same point and $\vec{w}$ pointing from the head of $\vec{v}$ to the head of $\vec{u}$.
2) Find an equation for $\vec{w}$ in terms of $\vec{u}$ and $\vec{v}$.
3) Use the equation to find a relationship between the squared lengths of $\vec{u}$, $\vec{v}$, $\vec{w}$, where the squared length of a vector is $u^2 = |\vec{u}|^2 = \vec{u} \cdot \vec{u}$. Use the relation between the dot product and angle between two vectors to recover the Law of Cosines.
4) Show that the area of the triangle $A = |\vec{u} \times \vec{v}|/2$. Prove the other cross products give the same result $A = |\vec{u} \times \vec{w}|/2 = |\vec{w} \times \vec{v}|/2$.

Question 2. A simple vector space is given by triples (a, b, c) where a, b, c come from the scalar field $\{0, 1, 2\}$, where the addition and multiplication tables for the field are:

+	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

*	0	1	2
0	0	0	0
1	0	1	2
2	0	2	1

For example, add two vectors $(2, 1, 0) + (1, 2, 2) = (0, 0, 2)$ and multiply a vector by a scalar $2 * (1, 2, 0) = (2, 1, 0)$.

1) List all 27 possible vectors in this space.
2) Show that $\vec{e}_0 = (1, 0, 0)$, $\vec{e}_1 = (2, 1, 0)$, and $\vec{e}_2 = (0, 2, 1)$ are basis vectors that span the space.
3) What is the additive inverse for these vectors? That is find a scalar d such that $(a, b, c) + d*(a, b, c) = (0, 0, 0)$. Often d would be -1, but -1 is not one of the scalars $0, 1, 2$ available.

Question 3. A roller coaster follows the trajectory:

$$\vec{x}(t) = \left(vt, 0, v^2t^2(vt - 10\,m)(vt - 18\,m)/1260\,m^3\right),$$

for $(-5\,s \le t \le 20\,s)$, and $v = 1\,m/s$. $\vec{r} = (x, y, z)$ is shorthand for $\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$.

1) If the height $h(t) = \vec{x}(t) \cdot \hat{z}$, then when (t) and where (x, y, h) does the coaster reach the maximum and minimum heights for $(-5\,s \le t \le 20\,s)$?
2) What is the velocity vector at $t = -5\,s$, the maximum, and minimum heights?
3) What is the acceleration vector at $t = -5\,s$, the maximum and minimum heights?
4) Is the total energy, kinetic plus gravitational, conserved?

Question 4. A $m = 10\,\text{kg}$ model rocket produces an upward thrust of $F(t) = 49\,N/s^2(3\,s - t)t$ for $3\,s$. At $t = 0\,s$ the rocket is sitting at rest on the launch pad. Assume $g = 9.8\,m/s^2$ and that the mass of the fuel is negligible:

1) Draw a free body diagram of the rocket and pad at $t = 0$?
2) At what time t_{off} will the rocket leave the launch pad?
3) Write down a differential equation for the position of the rocket as a function of time $\vec{x}(t)$ for $0 \leq t < t_{off}$.
4) Draw a free body diagram at $t = t_{off}$.
5) Write down a differential equation for $\vec{x}(t)$ for $t_{off} \leq t < 3\,s$ and solve for $\vec{x}(t)$. What is the height of the rocket at $t = 3\,s$?
6) What is the velocity at $t = 3\,s$?
7) Use conservation of energy to find the maximum height that rocket reaches. For what times can we use conservation of energy?

Question 5. A $m = 20\,\text{kg}$ model rocket produces an upward thrust of $F(t) = 49\,N/s^2(5\,s - t)t$ for $5\,s$. At $t = 0\,s$ the rocket is sitting at rest on the launch pad. Assume $g = 9.8\,m/s^2$ and that the mass of the fuel is negligible:

1) Draw a free body diagram of the rocket and pad at $t = 0$?
2) At what time t_{off} will the rocket leave the launch pad?
3) Write down a differential equation for the position of the rocket as a function of time $\vec{x}(t)$ for $0 \leq t < t_{off}$.
4) Draw a free body diagram at $t = t_{off}$.
5) Write down a differential equation for $\vec{x}(t)$ for $t_{off} \leq t < 5\,s$ and solve for $\vec{x}(t)$. What is the height of the rocket at $t = 5\,s$?
6) What is the velocity at $t = 5\,s$?
7) Use conservation of energy to find the maximum height that rocket reaches. For what times can we use conservation of energy?