

Electron Diffraction

The City College of New York

Department of Physics

Apparatus:

Cathode ray tube with Al and graphite targets

Digital camera (Sony)

Introduction:

This experiment is designed to illustrate the wave nature of electrons and also the diffraction patterns created when a beam of monoenergetic electrons is incident on crystalline targets in the form of thin films.

The apparatus consists of a cathode ray tube in which a beam of electrons can be accelerated through a known potential difference to a given, controllable kinetic energy. The electron beam may be directed against any one of four crystalline targets which are in the form of thin films. Under proper conditions the target will scatter the beam only in certain preferred directions. The scattered electrons may be observed as they strike a phosphor coating on the inside of the front face of the cathode ray tube. From the positions of the spots or rings on the face of the tube and the distance from the thin film to the face of the tube, the angles through which the electron beam has been scattered may be determined. These angles are related to the crystal structure of the target material and the wavelengths of the electrons by the Bragg diffraction equation.

The two objectives of this experiment are:

- A. Confirmation of the de Broglie hypothesis concerning the wave nature of the electron, and, in particular, verification of the form of the relationship between the electron wavelength λ and momentum p , i.e., $\lambda = h/p$.
- B. Interpretation of the diffraction pattern of the scattered electrons to determine the crystal structure and lattice parameter of the aluminum thin film and comparison of these results with existing data. Information concerning the crystal structure of graphite can also be obtained.

Equipment and Operation:

Precautions: Do not exceed 10 microamperes electron beam current.

Operate between 5 and 10 microamperes only for short times.

Use the smallest current possible to observe the diffraction patterns.

Do not exceed 9.5 kilovolts accelerating potential.

Use the following procedure to obtain the diffraction patterns.

1. Turn the INTENSITY and VOLTAGE knobs to zero (ccw).
2. Turn the FOCUS knob to FULL (cw).
3. Turn the power switch (AC ON) and wait one minute for warm-up.
4. Turn VOLTAGE slowly to 7 kV.
5. Increase the current (INTENSITY) to 5 microamperes or until a spot or ring pattern appears on the screen, using whichever current is smaller. (The ammeter is placed on top of the apparatus.)
6. If the beam is not visible, adjust its position by means of the knobs marked HORIZ and VERT. As soon as the beam is visible, reduce the intensity to the smallest useable value.
7. Move the beam over the target and select a spot on the appropriate material. Graphite is at the upper left and aluminum covers the remaining three squares.
8. Focus the beam and reduce the intensity to the lowest useable value.

To turn the unit off, follow steps 1 and 2, wait for 1 minute and then turn the power off.

The distance from target to screen marked on the case of the CRT is 6.637 inches. The pattern on the face of the cathode ray tube may be photographed using a digital camera.

Experimental Procedure:

Part I

Show that the wavelength of a wave associated with an electron follows the prediction of de Broglie rather than, for example, the hypothesis that the pattern is due to the shortest wavelength x -rays that could be produced when the electrons strike the films. This may be accomplished by observing experimentally how the radius r of a given diffraction ring (from aluminum) depends on the accelerating potential V . The radius r should be measured for several values of V .

Since the de Broglie hypothesis relates the momentum p to the wavelength λ of the electron, it implies a relationship between the diffraction ring radius r and the accelerating potential V . You should derive this relationship and decide what sort of graph should be drawn to test de Broglie's hypothesis. You should also show how the data would appear on this graph if the alternate hypothesis, i.e., that the diffraction pattern is produced by x -rays, were true.

Part II

Bring a small magnet close to the screen. Does this indicate whether the diffraction pattern is due to electrons or x -rays?

Part III

Pick a suitable accelerating potential and record the diffraction pattern for Al using the digital camera. Measure the size of several rings of this diffraction pattern. Determine the lattice parameter a of aluminum from your data. Repeat for another wavelength. Note that aluminum is a face-centered cubic crystal for which diffracted beams are allowed only when the Miller indices (hkl) are all odd or all even. Use the de Broglie relation to determine the wavelength of the accelerated electrons. Does your calculated value of a agree with the standard value of $a = 0.405$ nm to within your estimate of the experimental uncertainty? It will be necessary to calculate plane spacings, i.e., $d(hkl)$ values, for the aluminum crystal structure.

Part IV

Obtain a single crystal diffraction pattern from graphite and record with the digital camera. Try to relate the symmetry of the pattern and angular relation of the spots to the structure of graphite.

Questions:

1. Should the relativistic formula $p = \frac{1}{c} \sqrt{E^2 - (mc^2)^2}$ for the momentum of an electron be used in your calculation? Note that $E = mc^2 + eV$ in this formula. The non-relativistic formula is $p = \sqrt{2mE}$ where $E = eV$. Justify your answer carefully.
2. Why are the rings in the diffraction pattern from graphite 'spotty'? Why do the spots shift when you shift the position of the beam slightly?
3. Why do rings appear in the diffraction pattern of aluminum only for Miller indices (hkl) that are all even or all odd and not for mixed sets of indices?

References:

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Barrett and Massalski, *Structure of Metals*, McGraw-Hill, 1966

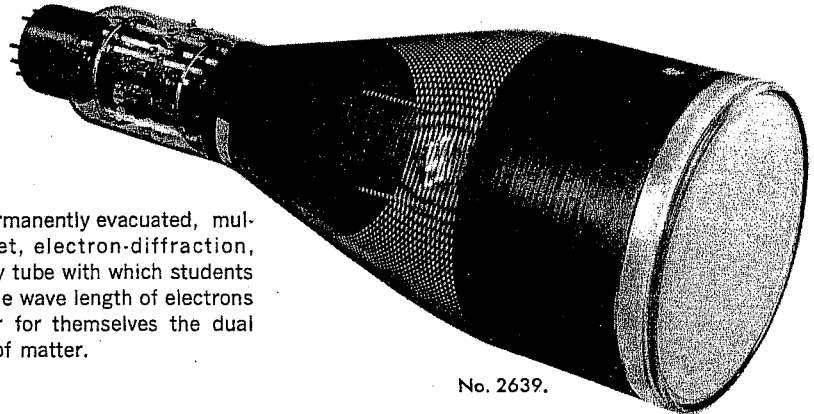
revised: 6/03

Catalog No. 2639

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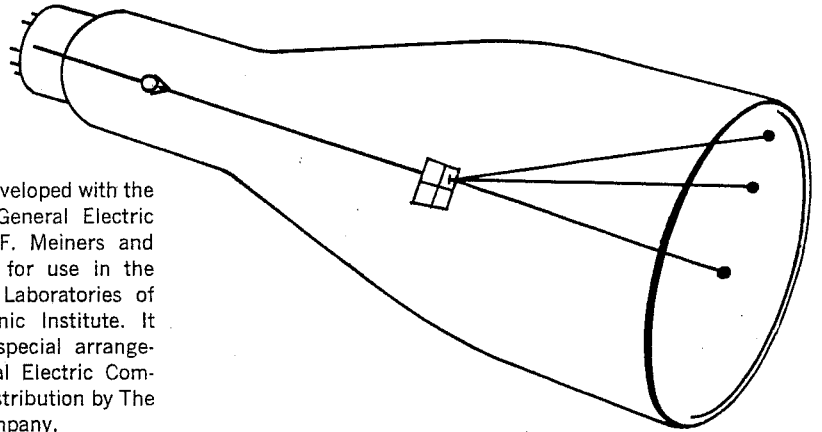
Electron Diffraction Tube

and Catalog No. 2639A Power Supply



A permanently evacuated, multiple-target, electron-diffraction, cathode-ray tube with which students measure the wave length of electrons to discover for themselves the dual character of matter.

No. 2639.



The tube was developed with the cooperation of the General Electric Company by Harry F. Meiners and Stanley A. Williams for use in the Sophomore Physics Laboratories of Rensselaer Polytechnic Institute. It is produced under special arrangement by The General Electric Company for exclusive distribution by The Welch Scientific Company.



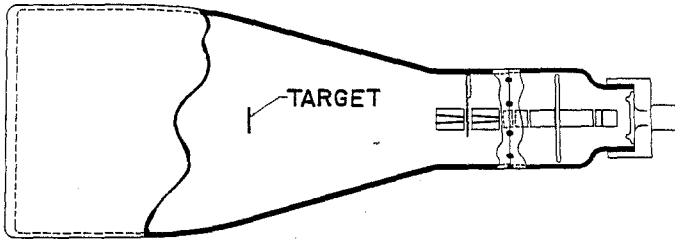
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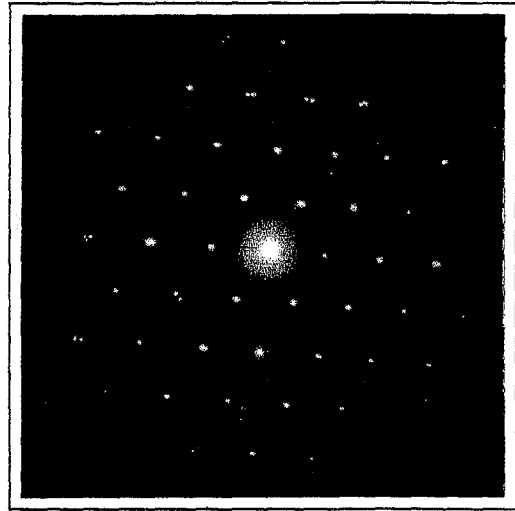
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Electron-Diffraction Cathode-ray Tube



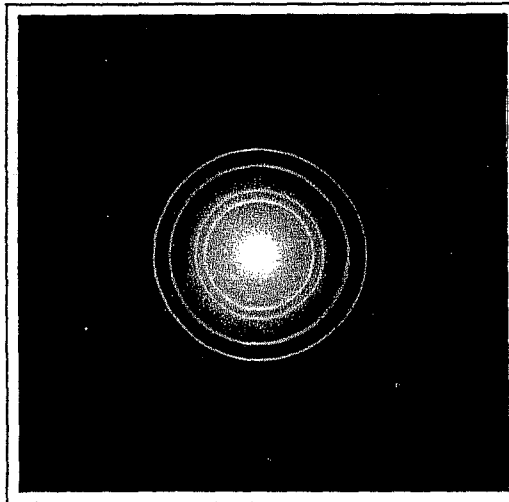
Two different target materials are provided to double usefulness of the tube.

TWO-DIMENSIONAL HEXAGONAL
PYROLYTIC GRAPHITE



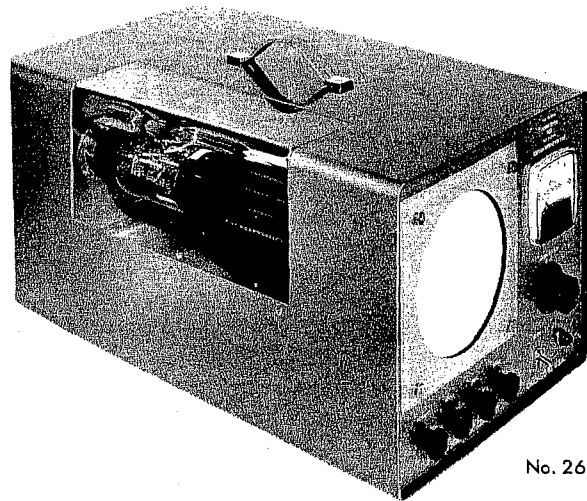
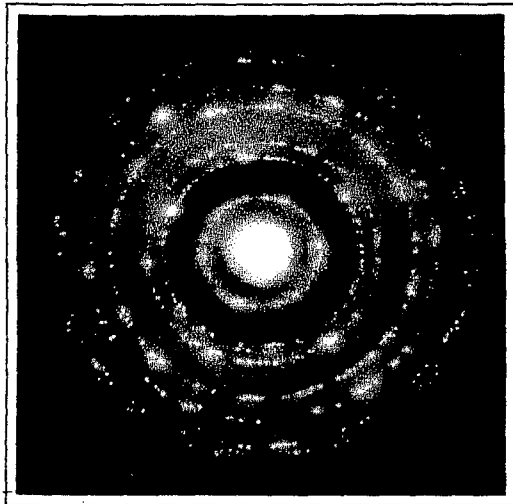
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2639A POWER SUPPLY, For
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using the same relationship that held for light, namely

$$\lambda = \frac{h}{p}, \quad (1)$$

where λ is the wavelength of a light wave, h is Planck's constant and p is the momentum of the photons. De Broglie proposed that matter, as well as light, has a dual character behaving in some circumstances like particles and in others like waves. He suggested that a particle of matter having a momentum p would have an associated wavelength λ .

The first experimental evidence of the existence of matter waves was obtained by Davisson and Germer² in 1927. They "reflected" slow electrons from a single crystal of nickel and applied de Broglie's relationship (See Eq. 4). The wavelength of the electrons was determined and compared with that calculated from Bragg's³ expression (See Eq. 5). Excellent agreement was obtained.

The wavelength of a beam of electrons, as indicated above, is given by

$$\lambda = \frac{h}{p} = \frac{h}{mv},$$

where (mv) is the momentum of a moving electron. The wavelength is therefore inversely proportional to the velocity v of the electrons. This velocity can be obtained directly from the accelerating potential V in the vacuum tube. Since the kinetic energy of the electrons is given by

$$\frac{1}{2} mv^2 = eV \quad (2)$$

the wavelength

$$\lambda = \frac{h}{\sqrt{2meV}} \quad (3)$$

where m is the mass and e is the charge of the electron. When the values of h , m , and e are substituted, this becomes

$$\lambda = \sqrt{\frac{150}{V}} \quad (4)$$

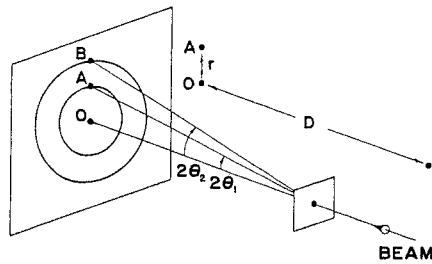
if λ is expressed in angstroms and V in volts. Since our apparatus is operated in general below 10 KV, electron energies are non-relativistic and no relativistic correction factor is needed.

The results of Thomson's⁴ experiments with fast electrons supplied additional evidence about the behavior of electrons. Thomson analyzed photographically diffraction patterns produced by electron beams passing through thin films of gold, aluminum and other materials. From measurements of the size of the electron diffraction rings on a fluorescent screen, the wavelength of the electrons was found and again agreed with that predicted by de Broglie's equation. Other experimental work has shown that all particles have dual properties.

1. L. de Broglie, Phil. Mag. 47, 446 (1924)
2. C. J. Davisson and L.H. Germer, Phys. Rev. 30, 705 (1927)
3. W. L. Bragg, Proc. Camb. Phil. Soc. 17, 43 (1913)
4. G. P. Thomson, Proc. Roy. Soc. 117, 600 (1928); 119, 652 (1928)

$$2 d \sin \theta = n \lambda^* \quad (5)$$

In this equation d is the separation between lattice planes, λ^* is the wavelength of the electrons, θ is the ordinary angle between the incident beam and the reflecting plane, which is the same as the angle between the reflected beam and the reflecting plane, and n is the order of the reflection. The angle between the direct and diffracted beams is 2θ , as shown in Fig. 1.



1. $2\theta = \frac{r}{D}$
2. $2a^* \sin \theta = (h^2 + k^2 + l^2)^{\frac{1}{2}} \lambda^* = 2d\theta = \frac{ar}{D}$
3. $\lambda^* = \frac{ar}{D} \frac{1}{(h^2 + k^2 + l^2)^{\frac{1}{2}}}$

Fig. 1. Ring diffraction pattern produced by the constructive interference of the electron waves diffracted from the various families of planes within the randomly oriented crystals in the thin film aluminum target.

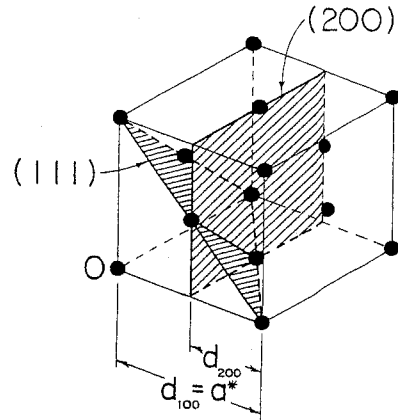


Fig. 2. The face-centered cubic lattice of aluminum. Two possible reflecting planes are shown. The separation between the (100) planes is called the lattice constant ($d_{100} = a^*$).

The separation d between reflection planes⁵ for a face-centered cubic structure (FCC), such as aluminum, in terms of the Miller indices (HKL) is

$$d = \frac{a^*}{\sqrt{H^2 + K^2 + L^2}} \quad (6)$$

where a^* is the length of the edge of the unit cell (Fig. 2). When the Miller indices are multiplied by the order of the reflection n , any order n of Bragg reflection from planes (HKL) is considered to be first order Bragg reflection from planes $[nH \ nK \ nL]$. Therefore,

$$d = \frac{na^*}{\sqrt{h^2 + k^2 + l^2}} \quad (7)$$

5. G. P. Harnwell and J. J. Livingood, Experimental Atomic Physics, McGraw-Hill Book Co. (1961)

When the angle θ in Eq. 8 is small, the $\sin \theta$ can be replaced with

$$\theta = \frac{r}{2D}, \quad (9)$$

and Eq. 8 becomes

$$\lambda^* = \frac{a^* r}{D} \cdot \frac{1}{\sqrt{b^2 + k^2 + l^2}}, \quad (10)$$

where D is the distance from target to screen and r is the radii of rings. a^* is known from x-ray measurements. D and r are obtained by direct measurements.

When comparing the wavelength calculated from de Broglie's relationship with that calculated from Bragg's expression, it is helpful to designate the two values as λ and λ^* , respectively, so that they can be tabulated without confusion. λ and λ^* represent the wavelength of the same electrons.

The observed diffraction pattern consisting of rings of various radii is produced by the constructive interference of the electron waves diffracted from the various families of planes within the randomly oriented crystals in the thin film target. The intensity of a reflection in the diffraction pattern is proportional to the square of the corresponding structure factor, i. e.,

$$I(bkl) \propto [F(bkl)]^2.$$

For a face-centered cubic structure,⁶

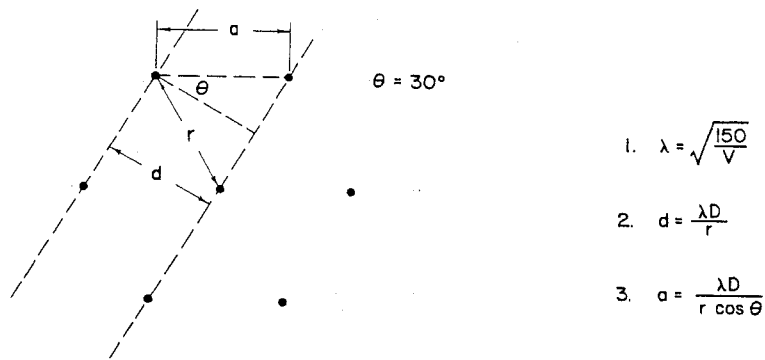
$$F(bkl) \sim 1 + e^{i\pi(b+k)} + e^{i\pi(k+l)} + e^{i\pi(l+b)}. \quad (11)$$

The structure factor actually takes into consideration the coordinates and differences in scattering power of the individual atoms, the Miller indices bkl , and the addition of sine waves of different amplitude and phase but of the same wavelength. When squared, the structure factor vanishes unless b, k, l are all odd or even, in which case $F \sim 4$. (See Table I for allowed FCC reflections for aluminum.) Therefore rings will occur for which b, k, l are all odd or even.

TABLE I - ALLOWED FCC REFLECTIONS FOR ALUMINUM

bkl	$(b^2 + k^2 + l^2)$	$(b^2 + k^2 + l^2)^{\frac{1}{2}}$
111	3	1.732
200	4	2.000
220	8	2.828
311	11	3.316
222	12	3.464
400	16	4.000
331	19	4.358
420	20	4.472
422	24	4.898
511, 333	27	5.196
440	32	5.656

6. C. Kittel, Introduction to Solid State Physics, John Wiley and Sons, Inc. (1957)



1. $\lambda = \sqrt{\frac{150}{V}}$
2. $d = \frac{\lambda D}{r}$
3. $a = \frac{\lambda D}{r \cos \theta}$

Fig. 3. Analysis of hexagonal spot diffraction pattern. The second equation is derived from Bragg's relationship. d is the separation between planes. D is the distance from target to screen. The distance r from the central spot to each of the spots on the hexagon is measured. a is the calculated lattice constant which is compared to the known a^* found by x-ray measurements.

To obtain the separation d between planes,⁹ first calculate the wavelength λ of the electrons from de Broglie's relationship (Eq. 4) and substitute in Bragg's relationship, λ for λ^* . For small angles the latter becomes

$$d = \frac{\lambda D}{r} \quad (12)$$

Once d is determined, the lattice constant a for hexagonal pyrolytic graphite can be calculated and compared with the known constant a^* obtained from x-ray measurements. Conversely, given a^* , the wavelength λ^* can be calculated and compared with λ computed from the voltage.

OPERATING PROCEDURE

Warning! Do not touch connecting wires at the rear of the power supply or tube. High voltage is dangerous. If for any reason the tube must be moved or wires disconnected, call your instructor. The tube is evacuated to a pressure of 10^{-8} mm Hg and should be handled with caution to avoid implosion.

1. A microammeter should be connected to the external meter jack on the power supply and used at all times to measure target currents. The range will depend upon which application is being used, as indicated below. (The required meter is assumed to be available in any physics laboratory and is therefore not included with the power supply.)

7. P. P. Ewald, Z. Krist. 56, 129 (1921)

8. M. von Laue, Ann. d. Phys. 29, 211, (1937)

9. See G. Thomas, Transmission Electron Microscopy of Metals, John Wiley and Sons, New York, p. 30 or Harnwell and Livingood, p. 167.

3. Using for demonstrations. A target current of 50 to 100 microamperes is suggested. Continued operation at high current may be expected to shorten the life of the tube.
4. Using for closed-circuit television. Target currents of 10 to 25 microamperes should be adequate but will depend upon the type of TV equipment being used and will require some testing to get optimum conditions. The phosphor on the tube screen was selected so that both ring and spot diffraction patterns could be picked up with maximum effect using relatively low target currents. Tube and camera should be shielded somewhat from room illumination, with black paper as a hood for example.
5. Before turning on the power supply, turn the intensity (bias) and voltage controls to "off".

Turn the A.C. line switch on. Allow a few minutes for the power supply and the tube filament to warm up. Then turn the high voltage control to the desired value.

6. To avoid burning the phosphor screen, turn the intensity (or bias) control on slowly. Do not exceed current ratings. Do not at any time increase the intensity beyond the point where the power supply overloads and the high voltage meter dips. Watch the central beam spot for possible burning of the screen due to prolonged high intensity at one position. This will not occur if the centering controls are used to move the beam slowly while searching for diffraction patterns. If suggested target current ratings are followed, screen will not be burned when beam is focused on a particular target.
7. Always adjust focus control until spot size on screen is as small as possible for each selected high voltage reading.
8. Turn the intensity and voltage controls to "off" position before switching off A.C. line voltage.

SUMMARY

This tube will last a long time if it is used properly. Instructors should become thoroughly familiar with its characteristics before allowing students to take data. Always work with the minimum current needed to give good patterns for the purpose desired. Keep the spot moving slowly while searching for a good target. When the beam is focused on a target, intensity can be increased without screen damage. Correct the voltage after using the centering and focusing controls since there is some interaction between controls. A much more expensive supply would be needed to eliminate all interaction.

1. Look up the value of the lattice constant a^* which is known from x-ray measurements.
2. Using a series of voltages to be suggested by the instructor, measure the radii of all observed circular rings at each voltage. So that the effects of ring distortion may be minimized, obtain an average of four measurements of the radius of each ring by measuring across four different diameters. Caution! Check the voltage continuously while making measurements or correcting focus. A.C. line voltage may vary and affect meter readings. Record all your measurements, including the voltages.
3. Bring a small magnet near the face of the tube. The diffraction pattern is deflected with little distortion demonstrating that the pattern is formed by electrons and not electromagnetic radiation. Use the magnet carefully. If the electron beam is deflected greatly, internal arcing might result.
4. Apply Bragg's relationship and calculate the wavelength λ^* of the electrons for all permitted reflections (See Table I) at each voltage selected in paragraph 2. For the same voltages compute from de Broglie's equation the wavelength λ . Record all reflections and the corresponding values of λ^* and λ .

Since some allowed reflections cannot be observed because of their low intensity, check carefully for an obvious lack of agreement between λ^* and λ . For example, if the choice of the Miller indices for a particular measurement of the radius r of a ring pattern results in an unusual spread between λ^* and λ , then try other combinations of the indices until good agreement between λ^* and λ is obtained.

5. Discuss your results. Explain why the ring diameters increase with decrease in voltage.

SUGGESTED DATA TABLE FOR EXPERIMENT 1

Lattice constant a^* , _____ from x-ray data.
Target to screen distance D , _____ marked on the tube.

Accelerating Potential (Volts)	Reflection Plane	$(h^2 + k^2 + l^2)^{\frac{1}{2}}$	\bar{r} (cm)	λ (A)	λ^* (A)	%
	111	1.732				
	200	2.000				
	220	2.828				
	311	3.316				

Note. Repeat table for each voltage selected. Include under \bar{r} the four measurements of the radius of each ring and the mean \bar{r} .

- Determine the number of atoms associated with a unit cell for a face-centered cubic lattice. Use the molecular weight of aluminum M , its density ρ , Avogadro's number N_0 , the number of atoms in the unit cell, and calculate the lattice constant a^+ .
- Substitute for λ^* in Bragg's relationship, each λ obtained from de Broglie's relation for the various voltages used in Experiment 1. Calculate the lattice constant a for the allowed combinations of reflections. Record all values of a and the mean \bar{a} . Also include the corresponding voltages.
- Compare the known value of the lattice constant a^* , obtained from x-ray measurements, with the mean \bar{a} , found from Bragg's relationship, and a^+ , calculated above in paragraph 1. Discuss.

SUGGESTED DATA TABLE FOR EXPERIMENT 2

Lattice constant a^+ _____ directly from unit cell.
 Lattice constant a^* _____ from x-ray data.
 Target to screen distance D _____.

Accelerating Potential (Volts)	Reflection Plane	$(h^2 + k^2 + l^2)^{\frac{1}{2}}$	r (cm)	λ (Å)	a (Å)
	111	1.732	1 _____ 3 _____ 2 _____ 4 _____ $\bar{r} =$ _____		
	200	2.000	1 _____ 3 _____ 2 _____ 4 _____ $\bar{r} =$ _____		
	220	2.828	1 _____ 3 _____ 2 _____ 4 _____ $\bar{r} =$ _____		
	311	3.316	1 _____ 3 _____ 2 _____ 4 _____ $\bar{r} =$ _____		

Note. Repeat table for each voltage selected.

1. Derive from Bragg's relationship an expression which can be used to calculate the wavelength λ^* of the electrons. The equation should include the known lattice constant a^* of hexagonal pyrolytic graphite which is 2.456 Å obtained from x-ray measurements.
2. For a series of voltages to be recommended by the instructor, measure the distance r from the central beam spot to each of the spots in the hexagonal diffraction pattern. Caution! Check the voltage continuously while making measurements or correcting focus. Record each measurement r and the mean \bar{r} for the corresponding voltages.
3. Use the mean \bar{r} of the spot measurements to calculate the wavelength λ^* of the electrons. Also compute the wavelength λ by applying de Broglie's relationship for each voltage. Record λ^* and λ .
4. Does the agreement between λ^* and λ fall within the limits of accuracy of the apparatus? Discuss.

SUGGESTED DATA TABLE FOR EXPERIMENT 3

Lattice constant a^* _____ from x-ray data.
Target to screen distance D _____.

Accelerating Potential (Volts)	r (cm)	d (Å)	λ^* (Å)	λ (Å)	%
	1 _____ 4 _____				
	2 _____ 5 _____				
	3 _____ 6 _____				
	$\bar{r} =$ _____				

Note. Repeat table for each voltage selected.

Experiment 4. Calculation of the Lattice Constant of Pyrolytic Graphite

1. Substitute for the wavelength λ^* in Bragg's relationship λ obtained from de Broglie's equation. Compute the lattice constant a . Do this for each voltage selected in Experiment 3 or for a series of voltages recommended by your instructor. If voltages other than those used in Experiment 3 are selected, measure the distance r from the central beam spot to each of the spots in the hexagonal diffraction pattern. Be sure to check the voltage continuously while making measurements or correcting the focus. Record each measurement r and the mean \bar{r} for the corresponding voltages. Also record the calculated lattice constant a for each voltage.

SUGGESTED DATA TABLE FOR EXPERIMENT 4

Lattice constant a^* _____ from x-ray data.
 Target to screen distance D _____.

Accelerating Potential (Volts)	r (cm)	d (Å)	λ (Å)	a (Å)	%
	1 _____ 4 _____ 2 _____ 5 _____ 3 _____ 6 _____ $\bar{r} =$ _____				

Note. Repeat table for each voltage selected.

QUESTIONS

1. Prove that $\lambda(\text{Å}) = \sqrt{\frac{150}{V}}$.
2. Would the above equation be valid for a 10 Mev electron beam? If not, why not? Derive an equation which would apply.
3. How is the wavelength related to the electron's kinetic energy?
4. Assume that the electron beam is replaced with a beam of positively charged particles. Could you carry out the same analysis if the particles were positrons? What would be the result if the particles were protons?
5. Plot a graph of λ^2 vs $\frac{1}{V}$, where λ is in angstroms and V is in volts. Use data obtained from de Broglie's relationship. What is the significance of the slope?
6. Derive an expression for the separation d between planes for a simple cubic lattice.
7. Why is it necessary for the inner wall of the tube to be covered almost completely with a conducting coating, i. e., graphite?
8. Why does the diffraction pattern consist of concentric rings? Would a single crystal produce the same pattern? Explain.
9. Why are the target, screen, and conducting coating on the inner wall of the tube connected to ground?
10. Assuming that Helmholtz coils are placed on each side of the tube, derive an expression based on pattern deflection which could be used to calculate the speed v of the electrons. Also derive in terms of the accelerating voltage an alternate

mechanics.

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